Computer-Aided Optimization of Aircraft Structures

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This paper describes the principal methods used within the computer program STARS for the computer-aided design of optimum structures subject to a variety of constraints. In particular, the development of a Newton method for size optimization and a hierarchical approach to shape optimization are outlined. A test problem connected with the latter is presented. Practical examples are given that show how research that originated at the Royal Aerospace Establishment (RAE) has been continued and applied at Deutsche Airbus GmbH (DA) [formerly Messerschmitt-Bölkow-Blohm GmbH (MBB)]. This includes the design of various components that are typical in aircraft construction and also a description of the manner in which flutter optimization is being accomplished with STARS and DA in combination with the in-house aeroelastic program.

Nomenclature

c_i = bound on constraint g_i
g_i = behavioral constraint
L = Lagrangian function
m = number of active constraints
p = shape parameter
W = structural mass (weight)
w_j = component mass
x_j = design variable
z_j = reciprocal design variable, primal variable
δz_j = step in z_j
$\delta\lambda_i$ = step in λ_i
λ_i = Lagrange undetermined multiplier, dual variable
∇ = gradient operator
∇^2 = second derivative operator

I. Introduction

THE design of structural components is an iterative process in which the aim is to achieve a structure that is adequate in strength and stiffness, favorable to manufacture, and inexpensive: that is, in some sense, an optimum design. The design procedure can take a very long time if approached conventionally, and it is unlikely that components will in fact be optimized in detail against all important criteria.

The intensive use of computer methods, involving finite element (FE) codes together with computer-aided design (CAD) systems and finite element pre- and postprocessors, has provided an important step toward shortening the design process, and structural optimization provides a further valuable aid in this context. Time and cost benefits have been found from using structural optimization for the following: 1) weight assessment of design using various constructions and materials at a preliminary dimensioning stage; it is then that essential decisions are taken with regard to shaping of components or assemblies; 2) modification of structures, often at short notice, in the case of changes of specification, load changes, or the occurrence of flutter; and 3) further weight reduction prior to production.

References 1–4 contain general information on the state-of-the-art in structural optimization. The work described here relates to the Royal Aerospace Establishment (RAE) structural optimization program STARS, which primarily addresses size optimization problems based on simultaneous static, natural frequency, forced response, and flutter analysis. This paper outlines the principal methods used in the program and shows how the research originated at RAE has been continued and applied at Deutsche Airbus GmbH (DA).

II. Principal Methods

A. Foundations

STARS addresses the optimization of large structural models whose analysis is based on the finite element method. The size of the system of equations is such that analyses must be called upon extremely sparingly. The optimization seeks to minimize the structural weight with respect to the individual cross-sectional areas of bars or, correspondingly, the thicknesses of plates, while satisfying behavioral constraints on stress, displacement, local buckling, natural frequency, amplitude of vibration, flutter, and divergence speeds.

To be efficient, an optimization method requires knowledge of design sensitivities of the constraints with respect to these variables. STARS maintains a tight active-set strategy and, therefore, requires relatively few sensitivities to be calculated at any iteration. Fully analytic derivatives are used, to be contrasted with the semianalytic approach employed with NASTRAN, and the calculation employs the adjoint, or pseudoload, method.[5,6] Semianalytic sensitivities calculated using the direct method are, however, used to provide the higher order derivatives used for step length control and an approximate analysis capability.

Even to reduce the size of the optimization problem to the order of 50–1000 design variables requires the use of devices such as design variable linking. The use of design variable linking also has further advantages, such as providing the means to impose symmetry or fabrication requirements or to embody the designer's insight and prior experience. In addition, apart from problems involving simple bars or beams that can be modeled exactly, failure to employ design variable linking correctly will lead to false solutions in which the optimized structure is not correctly modeled by the analysis mesh. In such a case, there is a danger that optimization will merely serve to increase analysis errors. Hence, elements are treated in groups, each linked to a single design variable x_j by a Boolean matrix. In order to improve the linearity of constraints arising from the static analysis, the algorithms also use reciprocal variables $z_j = 1/x_j$ to define the design space.

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A variety of optimization methods are available within STARS, including fully stressing, optimality criterion, Newton, linear programming, and dual quadratic programming algorithms. These are best derived from the Lagrangian function

$$L(z,\lambda) = W(z) + \sum_{i=1}^{m} \lambda_i (g_i - c_i) \tag{1}$$

where $W(z)$ is the objective function and $g_i \leq c_i$ are the behavioral constraints.

The Lagrangian function depends on two sets of variables: the primal variables z_i and the dual variables λ_i (otherwise known as Lagrange undetermined multipliers). A necessary condition for the minimization of the original constrained optimization is that the Lagrangian function should be stationary with respect to both primal and dual variables. Differentiating yields the well-known Kuhn-Tucker conditions:

$$\frac{\partial W(z)}{\partial z_j} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial z_j} = 0 \qquad j = 1, \ldots, n \tag{2a}$$

$$\lambda_i (g_i - c_i) = 0 \qquad i = 1, \ldots, m \tag{2b}$$

$$\lambda_i \geq 0 \qquad i = 1, \ldots, m \tag{2c}$$

The location of a stationary point thus requires the solution of a system of $(n + m)$ simultaneously nonlinear equations, as difficult a task as the original minimization problem! Nonetheless, this Lagrangian form offers a variety of insights, both for the original primal problem and in providing the basis for deriving the dual problem, a maximization problem.[7]

Whichever method of optimization is adopted, the purpose remains the same, the satisfaction of the Kuhn-Tucker necessary conditions. The goal is to achieve this effectively and economically, particularly for the large structural system optimization problems where a vast number of design freedoms may exist. The very success of nonmathematical-programming techniques such as the stress-ratio method and optimality criterion methods show that it should be possible to make good progress.

B. Newton-Based Methods

As discussed elsewhere,[8] methods derived from considerations specific to the optimization of engineering systems, and owing little to classical mathematical programming techniques, led to fully stressing design and to optimality criterion methods. Each sets up formulas that may be applied iteratively to solve equations representing a subset of the Kuhn-Tucker conditions. In each instance, limitations to the applicability of the methods arise from the fact that the whole of the set of equations are not addressed simultaneously.

The goal of the STARS Newton method was to evolve a technique, based as rigorously as possible on mathematical concepts, but without losing the features that made the engineering-intuitive techniques work so well on large problems.

The emphasis was on complementing the optimality criterion method by providing estimates for the dual variables. It is assumed that the criterion constraints have been identified, and so the Kuhn-Tucker equations are formulated for a set of active, equality constraints. The initial selection of active constraints is somewhat heuristic, relying on fully stressing to select a set of stress constraints and on scaling to select the most critical stiffness constraints. Information from the linear program, used to provide a dual bound,[7] is then used to screen out many of these constraints by deleting those with zero dual variables.

As the first step toward solving the set of nonlinear simultaneous equations representing the Kuhn-Tucker conditions, a linear approximation is formed about the current point, giving equations

$$\left[\nabla^2 W + \sum_{i=1}^{m} \lambda_i \nabla^2 g_i\right] \delta z + \sum_{i=1}^{m} \nabla g_i \, \delta\lambda_i$$

$$+ (\nabla W = \sum_{i=1}^{m} \lambda_i \nabla g_i) = 0 \tag{3a}$$

$$\nabla g_i \, \delta z + (g_i - c_i) = 0 \tag{3b}$$

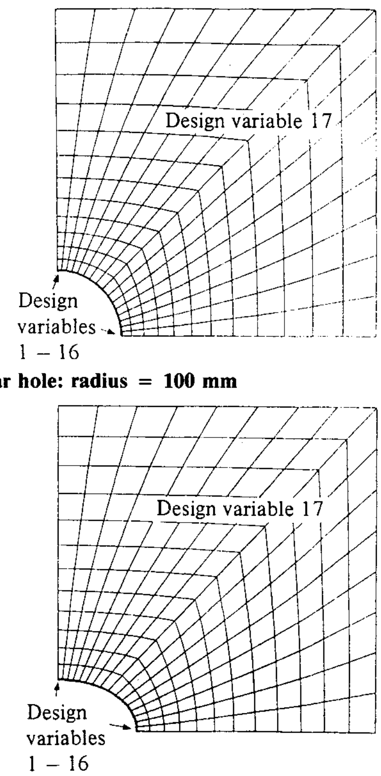
which determine the iteration step δz_i, $\delta\lambda_i$. For brevity, gradient and second partial derivative matrices have been denoted by ∇W and $\nabla^2 W$, and the summation over j representing the inner product with δz is not shown explicitly. Like any application of Newton's method, the repeated solution of this set of linear equations does not necessarily converge, but provided that the starting point lies within the domain of convergence, then that convergence will be quadratic. Unfortunately, despite recent advances in computational methods for the calculation of second derivatives,[9,10] the scale of the problem is such that this would still require excessive computation, and the storage for the second derivatives of all constraints in the active set, were it necessary, would require the use of tens or even hundreds of megabytes.

Thus, rather than employing an exact Newton step, the equations are further approximated by neglecting second derivatives of the constraints. Such approximations are already implicit in both the stress-ratio and optimality criterion methods and are known to be exact for statically determinate structures, optimized with respect to reciprocal variables z

$$\left[\frac{2w_j}{z_j^3}\right] \delta z + \sum_{i=1}^{m} \nabla g_i \, \delta\lambda_i - \left\{\frac{w_j}{z_j^2}\right\} + \sum_{i=1}^{m} \lambda_i \nabla g_i = 0 \tag{4a}$$

$$\nabla g_i \, \delta z + (g_i - c_i) = 0 \tag{4b}$$

Omitting the second derivatives of constraints, in fact, gives a very simple form for the linear equations. The weight as objective function is convex and separable, giving a diagonal



a) circular hole: radius = 100 mm

b) elliptic hole: semi-axes = 125 and 80 mm

Fig. 1 Meshes used for shape optimization.

Hessian matrix with positive coefficients. Thus, each of the set of equations [Eq. (4a)] may be used to eliminate one of the primal variables from Eq. (4b) explicitly, giving a reduced system of equations in which the dual variables are the unknowns. When these are found, they may then be substituted into the linearized optimality equations [Eq. (4a)]. The step taken by the Newton algorithm within STARS can be shown to be equivalent to an optimality criterion step combined with a weighted least-squares restoration step that moves into the tangent space of constraints.[8]

For a more general class of problem, this need to depart from the strict Newton form will lose the quadratic convergence properties; indeed, it is quite possible that the iteration may diverge from any solution. It is also shown[8] that this tendency may be compensated for by employing a step length control along a search direction lying in the tangent space and calculated using higher directional derivatives of the Lagrangian function. In practice, however, many structural problems appear to be exceptionally well behaved, giving good convergence to minimum weight designs with no such safeguards.

The Newton method is the primary technique used in the industrial applications that follow in Sec. IV. Before moving to such applications, it is shown how equations similar to Eqs. (4) are also obtained by considering the dependence of a size-optimized structure on an embedded parameter, representing geometric or material variation.

III. Shape of Optimization

To enable the large system optimization problems to be solved efficiently, considerable simplification of the design problem has been assumed: neither change of geometry nor material has been considered. As a first step to broaden the basis of the optimization, a one-parameter search over geometry has been considered. Rather than simply expanding the dimension of the design space, a hierarchical approach[11] has been adopted in which the parameter is used to move through a sequence of size-optimized designs. This both capitalizes on the achievement of efficient size optimization and enables the shape optimization to be terminated at any point with an efficient size-optimized design.

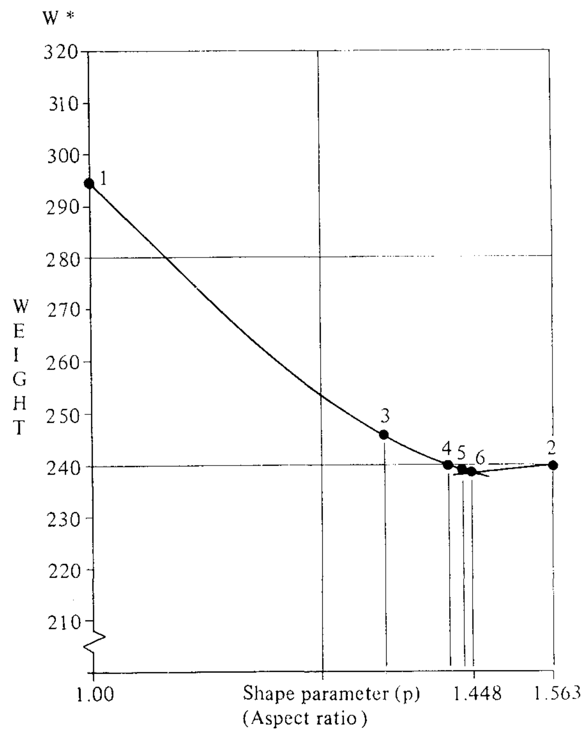


Fig. 2 Location of minimum weight for shape optimization.

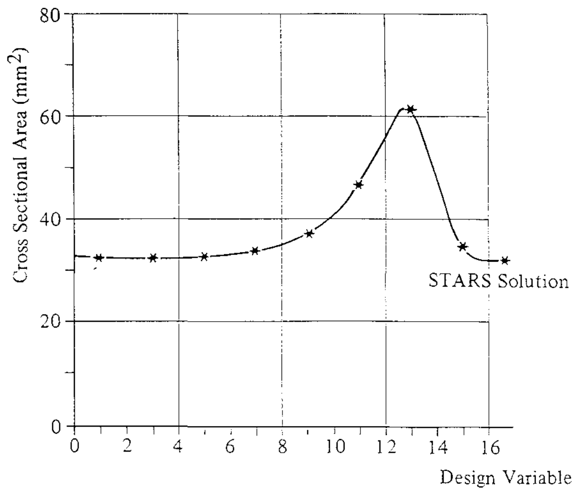


Fig. 3 Areas of edge reinforcement for normal mesh.

A. Theory

The method is based on variation of the Kuhn-Tucker necessary conditions that define the size optimum. The resulting equations, derived in Ref. 11, are similar in form to Eqs. (4) defining the Newton step itself, but these determine the sensitivity of the solution, dz/dp, $d\lambda/dp$, with respect to the parameter p. As for the Newton step, the equations are further approximated by neglecting second derivatives of the constraints, giving

$$\left[\frac{2w_j}{z_j^3}\right]\frac{dz}{dp} + \sum_{i=1}^{m} \nabla g_i \frac{d\lambda_i}{dp} + \left\{\frac{\partial W}{\partial p} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial p}\right\} = 0 \tag{5a}$$

$$\nabla g_i \frac{dz}{dp} + \frac{\partial g_i}{\partial p} = 0 \tag{5b}$$

This approximation again gives a very simple form for the linear equations but introduces some error into the estimate of sensitivity of the size of optimizing values. Nevertheless, the sensitivity of the objective function is accurate since, as Barthelemy et al.[12] show, given satisfaction of the Kuhn-Tucker conditions for the size optimization problem, any solution of Eq. (5b) will give

$$\frac{dW}{dp} = \frac{\partial W}{\partial p} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial p} \tag{6}$$

In use, weight sensitivities calculated from this simpler formula are slightly less accurate than those based on dz/dp due to errors in the estimates of the Lagrange multipliers. However, use of Eq. (6) paves the way to successful derivation, and subsequent use, of second derivatives d^2W/dp^2 (Ref. 12).

A further point of note is that slope discontinuities in dW/dp arise whenever a change of active set occurs in the underlying size optimum. The difficulties this causes, together with the cost of the semianalytic approach to calculating the partial derivatives with respect to p, make it unlikely that the shape facility within STARS will be extended beyond the one-parameter capability in the short term. Nonetheless, the one-parameter capability opens the way to automating a series of parametric studies, one of which is shown in the next section.

B. Test Problem

The test problem comprises a large sheet of uniform thickness, under a 2 : 1 biaxial load and contains a cutout of specified area. The cutout is taken to be an ellipse of unknown aspect ratio, and the sheet is reinforced by a variable-thickness bead, capable of carrying end load only, around its perimeter. Equilibrium considerations show the normal stress in the plate to balance the effect of the end load acting

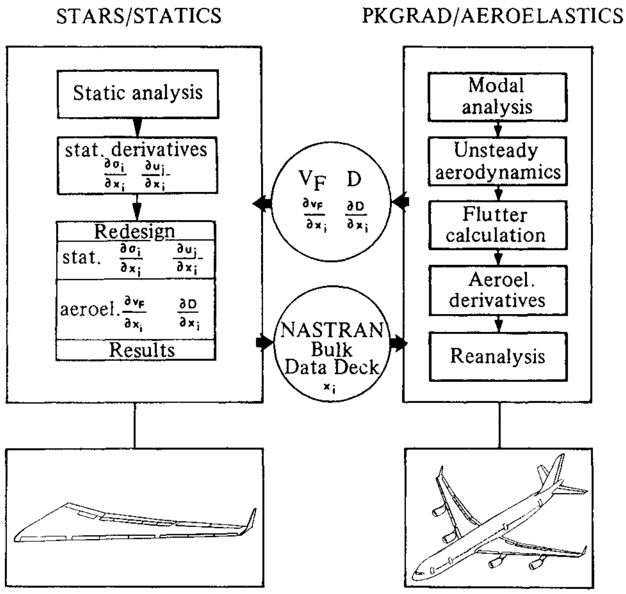


Fig. 4 Flutter optimization with STARS in a multimodel process.

through the curvature of the bead at that point, whereas the shear stress is in balance with the rate of change of end load in the bead around the perimeter.

A one-parameter family of meshes is created by interpolating nodal coordinates between a mesh on a sheet with a circular cutout and a similar mesh corresponding to an elliptic cutout of aspect ratio 2 : 1, see Fig. 1.

First, straightforward size optimization is performed for the extreme aspect ratios. In each case, selection of an optimum thickness variation of the reinforcement leads to a considerable reduction in the concentration of the von Mises equivalent stress, which was used as a strength criterion for the sheet material. Using the sensitivity calculation as the basis of linear or cubic interpolation of weight, the one-parameter shape optimization algorithm converged to the optimum in six steps, giving an aspect ratio of 1.45 : 1, as shown in Fig. 2. The variation in thickness of reinforcement around the cutout, corresponding to this aspect ratio, is shown in Fig. 3.

At the optimum design, although the stress components vary from point to point in the vicinity of the cutout, there is no concentration in the von Mises stress. A finer mesh was also tried to eliminate the possibility that the coarser mesh had missed a slight stress concentration, which would result in a considerably heavier solution if the sheet thickness had to be increased to compensate. The finer mesh, however, serves to substantiate the earlier run.

An alternative application of the one-parameter variation of size-optimized structure recently conducted is achieved by linking the parameter to material properties, in particular, the orientation of composite material. For example, this enables composite materials to be tailored to couple wing-bending loads and torsional deformation, in order to achieve static aeroelastic objectives in a forward swept wing.

Having considered small test problems used to validate the methods of structural optimization, we will now proceed with industrial problems where the major challenges lie both in the size of the problems addressed and in bridging the gap between theoretical and coding developments and practical engineering requirements.

IV. Industrial Usage of STARS

At DA, STARS is used for static, dynamic, and combined static-aeroelastic optimization.[13] STARS' modular structure makes it quite easy to incorporate additional modules (user-written software, FE programs, or pre- and postprocessors). This possibility is successfully used at DA. As far as the static side is concerned, MSC/NASTRAN, for example, is used in this manner both for analysis and for determination of the sensitivities required for optimization. This is of great practical importance because it enables stress engineers to use one and the same analysis program in every step of structural design work (from projecting stage to strength analysis, including structural optimization).

For the combined static-aeroelastic optimization, STARS and the DA in-house aeroelastic program PKGRAD[14] are modularly coupled. Figure 4 illustrates the systematics of this procedure for the flutter optimization process. The wing of an airliner has been taken as an example of a component that is to be optimized.

The basic idea is that the FE model of the entire aircraft is used for the aeroelastic analysis and computation of flutter derivatives, whereas only the FE model of the wing is used for the static analysis and determination of static derivatives.

Redesign in the course of flutter optimization is achieved with the Newton method (so-called pseudo-Newton method—PNM) described in Sec. II.A. First flutter optimization with this process, based on a submodel of the civil aircraft wing described in Sec. V has proved to be successful.

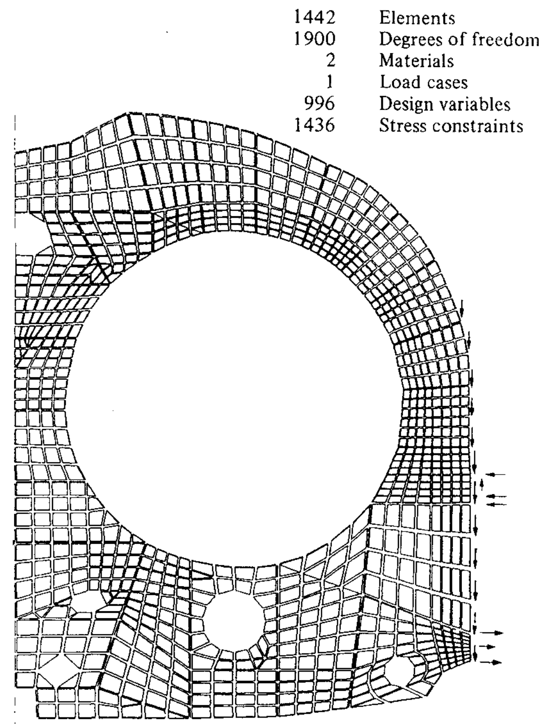


Fig. 5 FE model of frame.

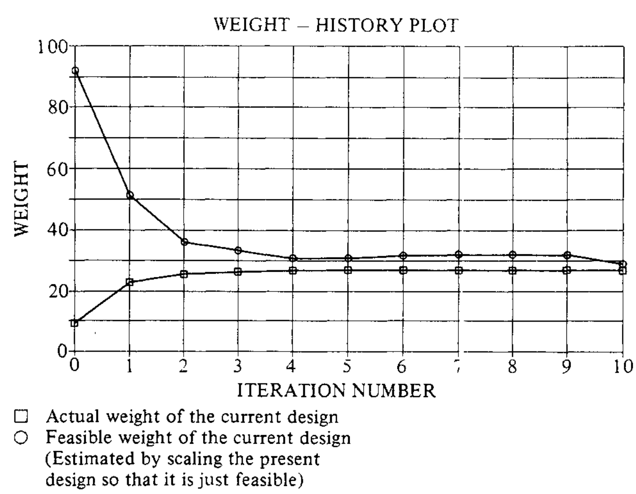


Fig. 6 Optimization results for frame.

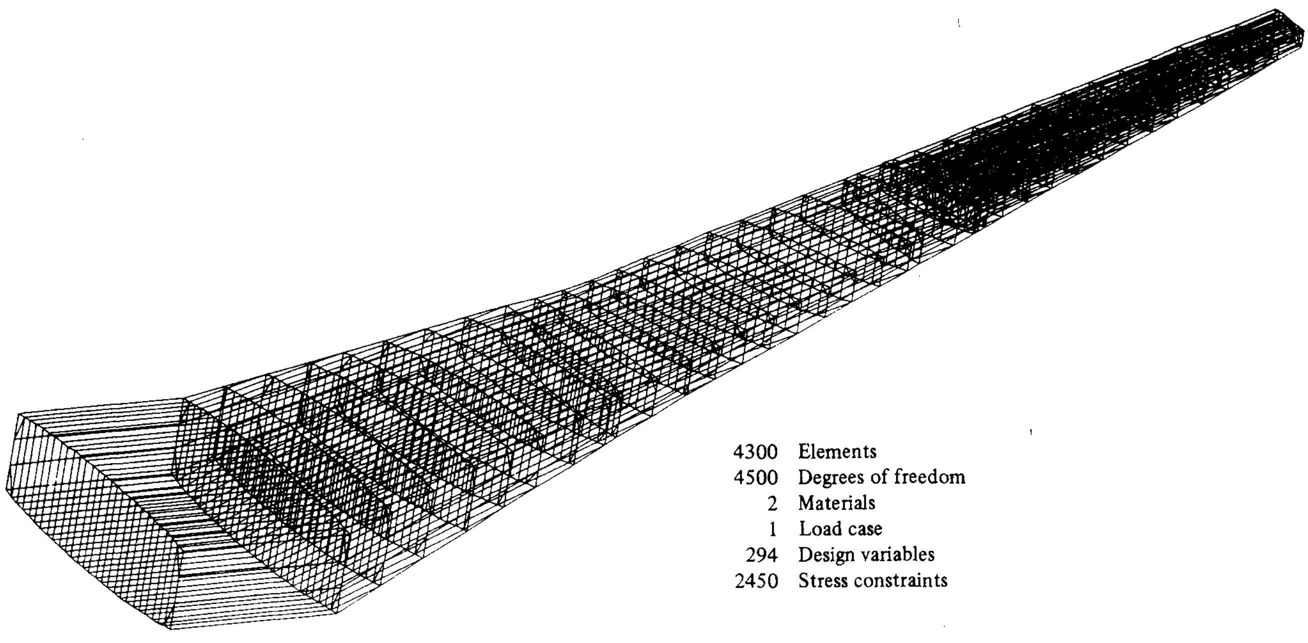


Fig. 7 FE model of wing box.

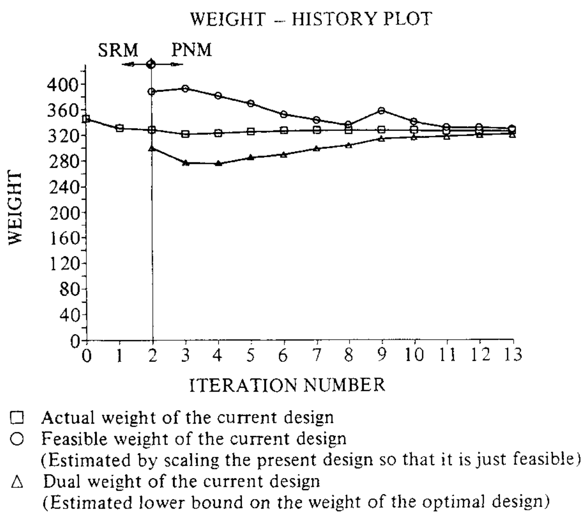


Fig. 8 Optimization result for wing box.

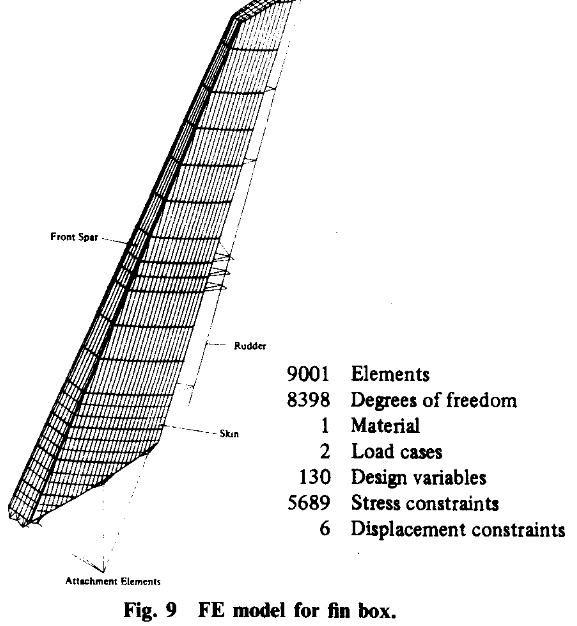


Fig. 9 FE model for fin box.

V. Industrial Applications

This section describes some characteristic examples of the practical application of STARS at DA. These examples relate to stress and stiffness optimization of metal and composite components.

In industrial application, it was found that time and cost advantages can be achieved particularly for the following fields if structural optimization is used: 1) predimensioning—early weight-optimal designs for various constructions and materials; 2) weight-optimal modification of structures at short notice in the case of specification changes, load changes, and occurrence of resonances, flutter, etc.; and 3) series producibility/value analysis.

It is very important to commence structural optimization during the preliminary-dimensioning phase since essential decisions with regard to the shaping of components or assemblies are made during this phase. Consequently, the detailing of FE models used for optimization will differ depending on the phase of application. The characteristic values of the FE models and optimization models are shown in the different illustration of the application examples.

A. Military Aircraft Frame

Figure 5 shows the FE model of the frame for a modern fighter aircraft. This aluminum frame is subjected to the wing attachment forces. From the optimization results obtained, Fig. 6 plots the weight curve vs the iterations. The weight curve shows clearly that the optimum weight is achieved within 10 iterations. In this case, the stress ratio method (SRM) was used as the optimization procedure for pure stress optimization. The minimum gauges for all finite elements to be optimized were specified as initial design.

B. Civil Aircraft Wing

Figure 7 gives a further practical example of the FE model of the wing box of a modern airliner. This FE model (metal inner wing/composite outer wing) served as the basis for stress optimizations at the inner and outer wing within the scope of a study. In these optimization runs, the number of design variables was systematically increased in successive runs. Figure 8 gives the example of a weight history plot for an optimization of the outer wing with 294 design variables.

The use of SRM was found to be effective for the woven fabric construction where longitudinal and transverse layers are linked to a common design variable. This was then followed by the more comprehensive PNM until the optimum

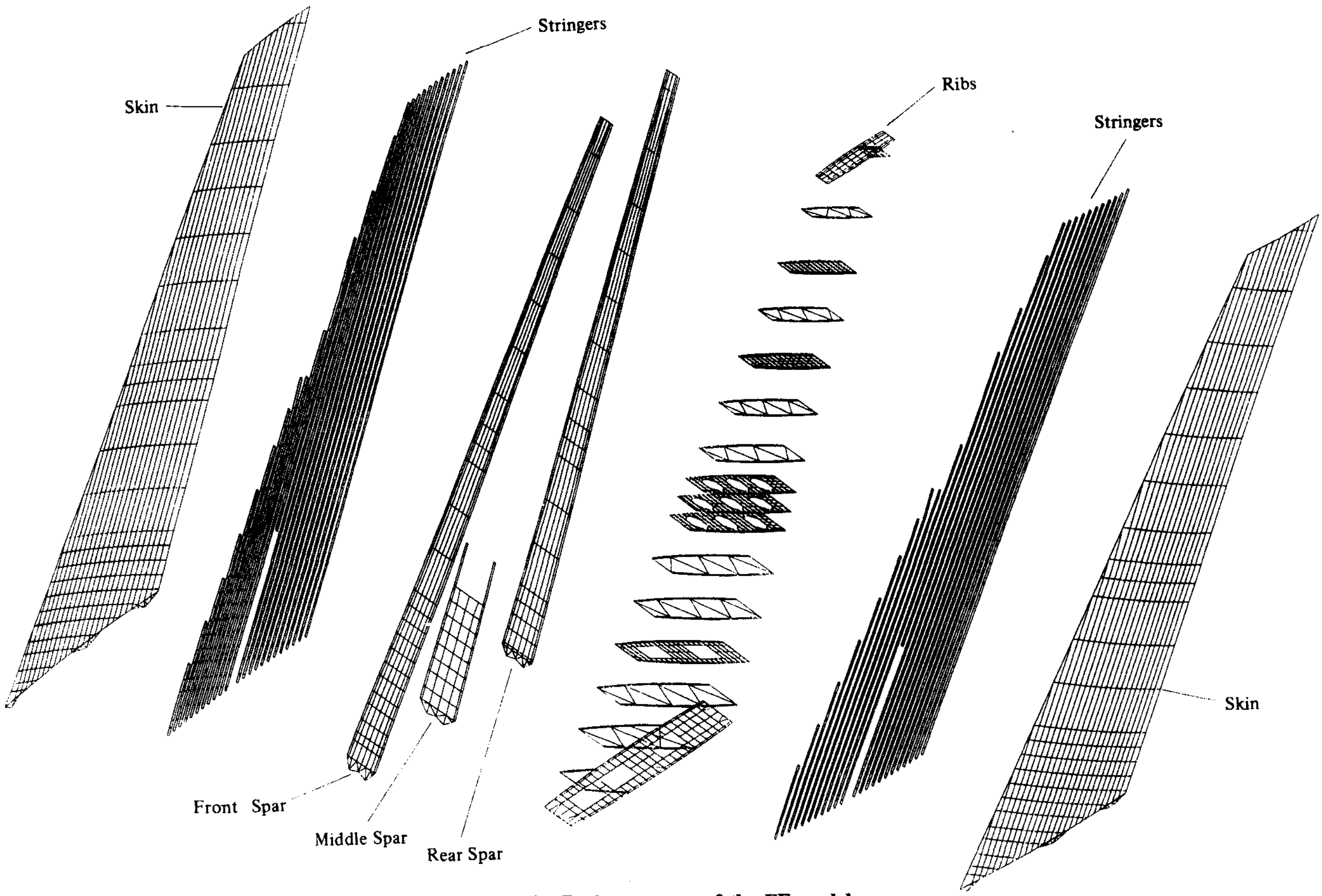


Fig. 10 Basic structure of the FE model.

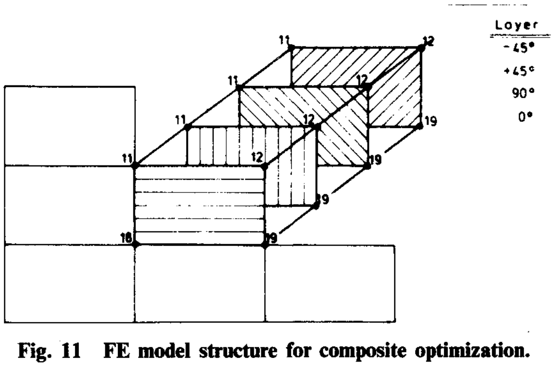


Fig. 11 FE model structure for composite optimization.

structure was reached. The curves show that the SRM quickly converges and reaches the almost optimum structure. The subsequent PNM introduces some corrections of the design, thus resulting in the optimum.

In practical industrial application, SRM and PNM have proven to be very successful for redesign purposes. Moreover, experience has shown that a combination of both procedures can be very advantageous for many practical problems.[13]

C. Airbus Tail Fin

Figure 9 shows the FE model of the composite fin box. The basic structure of the FE model is shown in Fig. 10.[15] In the composite design, layers with identical fiber direction were combined and idealized by a membrane element in the FE model (see Figs. 11 and 12). Weight optimization of the fin box was carried out for the two dimensioning load cases, lateral gusts, and maneuvers.

Allocation of the design variables to the element thicknesses and element cross-section of the individual component areas is shown in Fig. 13 for the skin, taking into consideration the following aspects:

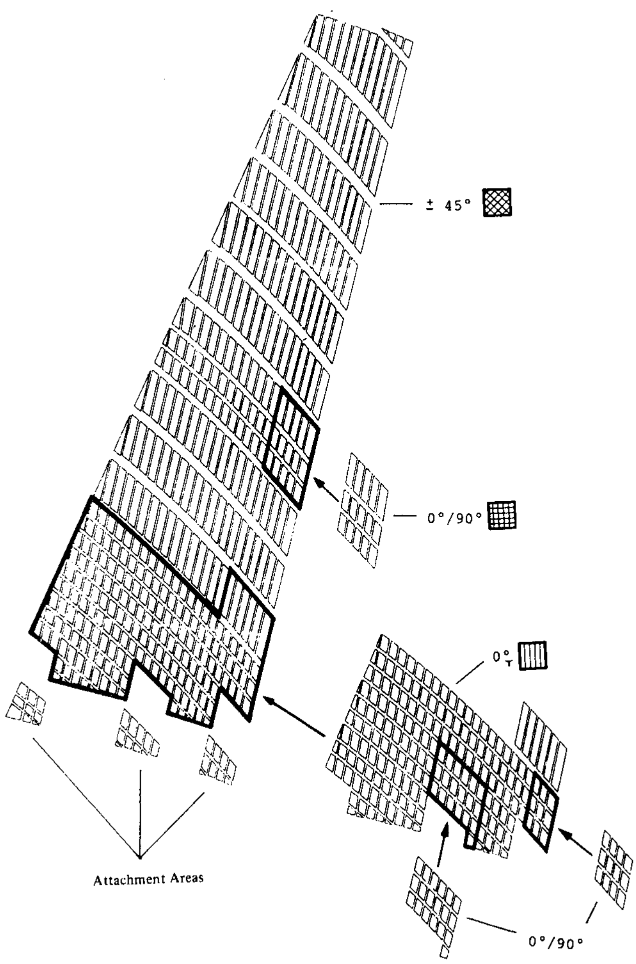


Fig. 12 Construction of skin FE model.

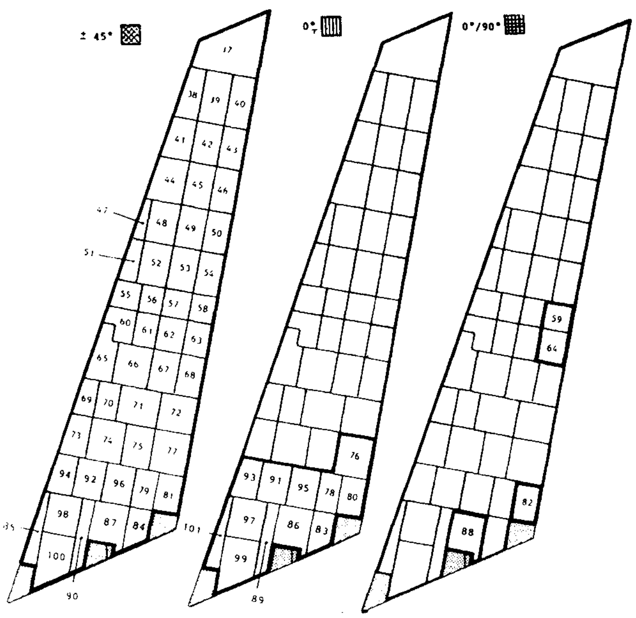
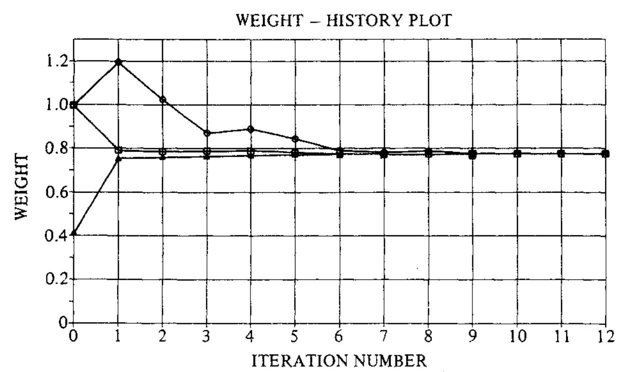


Fig. 13 Design variables of the skin.



□ Actual weight of the current design
○ Feasible weight of the current design
(Estimated by scaling the present design so that it is just feasible)
Δ Dual weight of the current design
(Estimated lower bound on the weight of the optimal design)

Fig. 14 Optimization results for fin box.

1) Since the 0/90- and ±45-deg layers of the skin are always a part of the same fabric and can thus be changed only as a whole, the thicknesses of the respective elements were linked to one design variable each (0 and 90 deg; +45 deg and −45 deg).

2) Owing to the symmetrical structure of the vertical tail, the respective left-hand and right-hand elements of the skin and of the stringers could be linked to one design variable each.

The following elements were fixed, i.e., they were not optimized: 1) connections of the fin box to the fuselage, 2) connection areas of skin and spars to the fuselage, 3) spar cap, 4) stringers in the connection areas just mentioned, and 5) all ribs.

Minimum limits of the element cross-sectional values to be optimized were governed by the specified structural and manufacturing requirements.

Displacement constraints at characteristic points of the front and rear spars and strain constraints for all elements to be optimized had been specified as constraints.

No detailed stability was carried out within the scope of optimization. A subsequent simplified analysis of local stability was performed for the optimized structure based on the existing strains. However, only strains parallel to the stringers were taken into account.

The pseudo-Newton method was selected as the optimization procedure since, in the present case, stress and displacement constraints are specified.

Figure 14 shows the standardized weight development vs the number of iterations as optimization results. It can be seen that 1) compared to the original component, which had been designed in the conventional manner, notable weight savings had been achieved; and 2) an optimum weight was achieved after the seventh iteration step, and virtually none of the constraints was violated any longer. This is indicated by the fact that the actual weight and feasible weight are practically equal from the seventh iteration onward.

The CPU time for seven iterations amounted to approximately 1.8 h on the scalar computer IBM 3090-400.

A subsequent simplified analysis of local stability based on strains resulted in the fact that the skin of the optimized structure had to be thickened in parts. The optimization described earlier applied to the idealized component structure. In this case the results and the resulting element cross sections and layer thicknesses were defined solely numerically. Therefore, adaptation of these values to the production requirements will again result in some increase of the optimized model weight.

This example of practical application for the carbon fiber reinforced plastic (CFRP) Airbus fin box shows that STARS can also be used for weight optimization of large structures under realisitc conditions.

VI. Conclusions

Industrial application of structural optimization at DA has convincingly demonstrated many advantages. Large and complex structural components made of metal and composite materials have been weight-optimized using the methods described in this paper, and considerable weight savings over conventional component design are possible. Structural optimization has become an efficient design tool in both preliminary and main design phases, making it possible to develop components at minimum weight and low cost within a relatively short time and thus to improve competitiveness.

Acknowledgments

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